GOALS

• Reconnaissance activity identification on the substation normally-open switches,

• Voltage stability index monitoring.

THEVENIN EQUIVALENT CIRCUIT

In the sequence domain, assuming transposed lines in the transmission level:

\[
\begin{bmatrix}
1 & 0 & 0 \\
-1 & 1 & 0 \\
-1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
\frac{\mathcal{R}_2}{j\omega}\mathcal{X}_2 & 0 & 0 \\
0 & \frac{\mathcal{R}_2}{j\omega}\mathcal{X}_2 & 0 \\
0 & 0 & \frac{\mathcal{R}_2}{j\omega}\mathcal{X}_2
\end{bmatrix}
\begin{bmatrix}
\mathcal{I}_2 \\
\mathcal{I}_2 \\
\mathcal{I}_2
\end{bmatrix} = \begin{bmatrix}
\mathcal{I}_0 \\
\mathcal{I}_0 \\
\mathcal{I}_0
\end{bmatrix}
\]

UNBALANCED GRID

• Taking advantage of unbalanced data

Assuming

\[
\mathcal{Z}_1(k) = \mathcal{Z}_2(k)
\]

\[
\mathcal{Z}_3(k) = \frac{\mathcal{R}_2}{j\omega}\mathcal{X}_2(k)
\]

BALANCED GRID

The only non-trivial equation is:

\[
\mathcal{I}_0(k) = \mathcal{E}_0(k) - \mathcal{Z}_0(k)\mathcal{I}_0(k)
\]

Assumption: The resistive part of the Thevenin impedance is negligible compared to the inductive part.

Let \( \mathcal{I}_k = [\mathcal{I}_k] \) and \( \mathcal{V}_k = [\mathcal{V}_k] \) be the imaginary component of the current, then:

\[
\begin{bmatrix}
\mathcal{A}(\mathcal{X}) - \mathcal{C}(\mathcal{X}) - \mathcal{X}(\mathcal{X})\mathcal{A}(\mathcal{X})
\end{bmatrix} = 0
\]

We form the following over-determined homogeneous set of equations:

\[
\begin{bmatrix}
\mathcal{A}(\mathcal{X}) - \mathcal{C}(\mathcal{X}) - \mathcal{X}(\mathcal{X})\mathcal{A}(\mathcal{X})
\end{bmatrix} = 0
\]

We form the following objective function:

\[
\mathcal{J}(\mathcal{X}_0, \mathcal{X}) = \frac{1}{2} \| \mathcal{A}(\mathcal{X}) - \mathcal{C}(\mathcal{X}) - \mathcal{X}(\mathcal{X})\mathcal{A}(\mathcal{X}) \|^2
\]

We formulate the Thevenin estimation problem as follows:

\[
\min_{\mathcal{X}} \mathcal{J}(\mathcal{X}_0, \mathcal{X})
\]

Where \( \mathcal{X} = [\mathcal{X}_0, \mathcal{X}]^T \)

Advantages:

• Having the assumption of constant Thevenin voltage phase angle over a window of M samples is not needed,

• reporting phasor angles relative to the voltage phasor angle removes the effect of off-nominal frequency.

The Levenberg-Marquardt Algorithm (LMA) is used to solve the non-linear least square problem because:

1) It handles better close to rank-deficient matrices, and 2) has better performance compared to Gauss-Newton for a bad initial guess.